

**90 Minutes—Scientific Calculator**

*Notes:* (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

1. The area of the region enclosed by the graphs of  $y = x^2$  and  $y = x$  is

- (A)  $\frac{1}{6}$                       (B)  $\frac{1}{3}$                       (C)  $\frac{1}{2}$                       (D)  $\frac{5}{6}$                       (E) 1
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2. If  $f(x) = 2x^2 + 1$ , then  $\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x^2}$  is

- (A) 0                      (B) 1                      (C) 2                      (D) 4                      (E) nonexistent
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3. If  $p$  is a polynomial of degree  $n$ ,  $n > 0$ , what is the degree of the polynomial  $Q(x) = \int_0^x p(t) dt$ ?

- (A) 0                      (B) 1                      (C)  $n - 1$                       (D)  $n$                       (E)  $n + 1$
- 

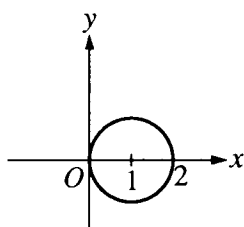
4. A particle moves along the curve  $xy = 10$ . If  $x = 2$  and  $\frac{dy}{dt} = 3$ , what is the value of  $\frac{dx}{dt}$ ?

- (A)  $-\frac{5}{2}$                       (B)  $-\frac{6}{5}$                       (C) 0                      (D)  $\frac{4}{5}$                       (E)  $\frac{6}{5}$

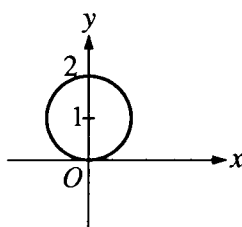
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5. Which of the following represents the graph of the polar curve  $r = 2 \sec \theta$ ?

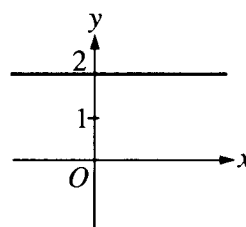
(A)



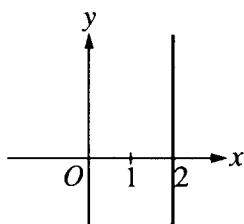
(B)



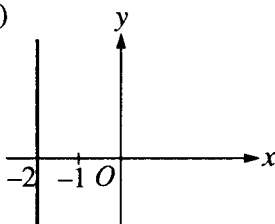
(C)



(D)



(E)



6. If  $x = t^2 + 1$  and  $y = t^3$ , then  $\frac{d^2y}{dx^2} =$

(A)  $\frac{3}{4t}$

(B)  $\frac{3}{2t}$

(C)  $3t$

(D)  $6t$

(E)  $\frac{3}{2}$

7.  $\int_0^1 x^3 e^{x^4} dx =$

(A)  $\frac{1}{4}(e-1)$

(B)  $\frac{1}{4}e$

(C)  $e-1$

(D)  $e$

(E)  $4(e-1)$

8. If  $f(x) = \ln(e^{2x})$ , then  $f'(x) =$

(A) 1

(B) 2

(C)  $2x$

(D)  $e^{-2x}$

(E)  $2e^{-2x}$

9. If  $f(x) = 1 + x^{\frac{2}{3}}$ , which of the following is NOT true?

- (A)  $f$  is continuous for all real numbers.
- (B)  $f$  has a minimum at  $x = 0$ .
- (C)  $f$  is increasing for  $x > 0$ .
- (D)  $f'(x)$  exists for all  $x$ .
- (E)  $f''(x)$  is negative for  $x > 0$ .

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10. Which of the following functions are continuous at  $x = 1$ ?

- I.  $\ln x$
- II.  $e^x$
- III.  $\ln(e^x - 1)$

- (A) I only    (B) II only    (C) I and II only    (D) II and III only    (E) I, II, and III

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11.  $\int_4^\infty \frac{-2x}{\sqrt[3]{9-x^2}} dx$  is

- (A)  $7^{\frac{2}{3}}$     (B)  $\frac{3}{2} \left( 7^{\frac{2}{3}} \right)$     (C)  $9^{\frac{2}{3}} + 7^{\frac{2}{3}}$     (D)  $\frac{3}{2} \left( 9^{\frac{2}{3}} + 7^{\frac{2}{3}} \right)$     (E) nonexistent

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12. The position of a particle moving along the  $x$ -axis is  $x(t) = \sin(2t) - \cos(3t)$  for time  $t \geq 0$ . When  $t = \pi$ , the acceleration of the particle is

- (A) 9    (B)  $\frac{1}{9}$     (C) 0    (D)  $-\frac{1}{9}$     (E) -9

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13. If  $\frac{dy}{dx} = x^2 y$ , then  $y$  could be

- (A)  $3 \ln \left( \frac{x}{3} \right)$     (B)  $e^{\frac{x^3}{3}} + 7$     (C)  $2e^{\frac{x^3}{3}}$     (D)  $3e^{2x}$     (E)  $\frac{x^3}{3} + 1$

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14. The derivative of  $f$  is  $x^4(x-2)(x+3)$ . At how many points will the graph of  $f$  have a relative maximum?

- (A) None                      (B) One                      (C) Two                      (D) Three                      (E) Four
- 

15. If  $f(x) = e^{\tan^2 x}$ , then  $f'(x) =$

- (A)  $e^{\tan^2 x}$   
(B)  $\sec^2 x e^{\tan^2 x}$   
(C)  $\tan^2 x e^{\tan^2 x - 1}$   
(D)  $2 \tan x \sec^2 x e^{\tan^2 x}$   
(E)  $2 \tan x e^{\tan^2 x}$
- 

16. Which of the following series diverge?

- I.  $\sum_{k=3}^{\infty} \frac{2}{k^2 + 1}$   
II.  $\sum_{k=1}^{\infty} \left(\frac{6}{7}\right)^k$   
III.  $\sum_{k=2}^{\infty} \frac{(-1)^k}{k}$

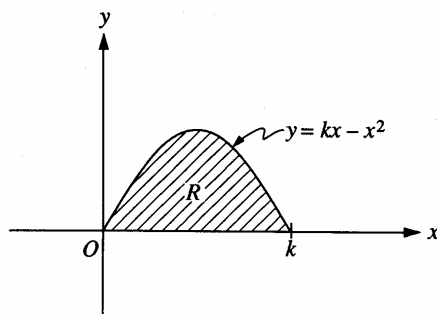
- (A) None                      (B) II only                      (C) III only                      (D) I and III                      (E) II and III
- 

17. The slope of the line tangent to the graph of  $\ln(xy) = x$  at the point where  $x = 1$  is

- (A) 0                      (B) 1                      (C)  $e$                       (D)  $e^2$                       (E)  $1 - e$
- 

18. If  $e^{f(x)} = 1 + x^2$ , then  $f'(x) =$

- (A)  $\frac{1}{1+x^2}$                       (B)  $\frac{2x}{1+x^2}$                       (C)  $2x(1+x^2)$                       (D)  $2x(e^{1+x^2})$                       (E)  $2x \ln(1+x^2)$
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19. The shaded region  $R$ , shown in the figure above, is rotated about the  $y$ -axis to form a solid whose volume is 10 cubic units. Of the following, which best approximates  $k$ ?

(A) 1.51                      (B) 2.09                      (C) 2.49                      (D) 4.18                      (E) 4.77

20. A particle moves along the  $x$ -axis so that at any time  $t \geq 0$  the acceleration of the particle is  $a(t) = e^{-2t}$ . If at  $t = 0$  the velocity of the particle is  $\frac{5}{2}$  and its position is  $\frac{17}{4}$ , then its position at any time  $t > 0$  is  $x(t) =$

(A)  $-\frac{e^{-2t}}{2} + 3$   
 (B)  $\frac{e^{-2t}}{4} + 4$   
 (C)  $4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$   
 (D)  $\frac{e^{-2t}}{2} + 3t + \frac{15}{4}$   
 (E)  $\frac{e^{-2t}}{4} + 3t + 4$

21. The value of the derivative of  $y = \frac{\sqrt[3]{x^2 + 8}}{\sqrt[4]{2x + 1}}$  at  $x = 0$  is

(A)  $-1$                       (B)  $-\frac{1}{2}$                       (C)  $0$                       (D)  $\frac{1}{2}$                       (E)  $1$

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22. If  $f(x) = x^2 e^x$ , then the graph of  $f$  is decreasing for all  $x$  such that

- (A)  $x < -2$       (B)  $-2 < x < 0$       (C)  $x > -2$       (D)  $x < 0$       (E)  $x > 0$

23. The length of the curve determined by the equations  $x = t^2$  and  $y = t$  from  $t = 0$  to  $t = 4$  is

- (A)  $\int_0^4 \sqrt{4t+1} \, dt$   
 (B)  $2 \int_0^4 \sqrt{t^2+1} \, dt$   
 (C)  $\int_0^4 \sqrt{2t^2+1} \, dt$   
 (D)  $\int_0^4 \sqrt{4t^2+1} \, dt$   
 (E)  $2\pi \int_0^4 \sqrt{4t^2+1} \, dt$

24. Let  $f$  and  $g$  be functions that are differentiable for all real numbers, with  $g(x) \neq 0$  for  $x \neq 0$ .

If  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x) = 0$  and  $\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$  exists, then  $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$  is

- (A) 0  
 (B)  $\frac{f'(x)}{g'(x)}$   
 (C)  $\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$   
 (D)  $\frac{f'(x)g(x) - f(x)g'(x)}{(f(x))^2}$   
 (E) nonexistent

25. Consider the curve in the  $xy$ -plane represented by  $x = e^t$  and  $y = te^{-t}$  for  $t \geq 0$ . The slope of the line tangent to the curve at the point where  $x = 3$  is

- (A) 20.086      (B) 0.342      (C) -0.005      (D) -0.011      (E) -0.033

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26. If  $y = \arctan(e^{2x})$ , then  $\frac{dy}{dx} =$

- (A)  $\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$  (B)  $\frac{2e^{2x}}{1+e^{4x}}$  (C)  $\frac{e^{2x}}{1+e^{4x}}$  (D)  $\frac{1}{\sqrt{1-e^{4x}}}$  (E)  $\frac{1}{1+e^{4x}}$

27. The interval of convergence of  $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$  is

- (A)  $-3 < x \leq 3$  (B)  $-3 \leq x \leq 3$  (C)  $-2 < x < 4$   
(D)  $-2 \leq x < 4$  (E)  $0 \leq x \leq 2$

28. If a particle moves in the  $xy$ -plane so that at time  $t > 0$  its position vector is  $(\ln(t^2 + 2t), 2t^2)$ , then at time  $t = 2$ , its velocity vector is

- (A)  $\left(\frac{3}{4}, 8\right)$  (B)  $\left(\frac{3}{4}, 4\right)$  (C)  $\left(\frac{1}{8}, 8\right)$  (D)  $\left(\frac{1}{8}, 4\right)$  (E)  $\left(-\frac{5}{16}, 4\right)$

29.  $\int x \sec^2 x \, dx =$

- (A)  $x \tan x + C$  (B)  $\frac{x^2}{2} \tan x + C$  (C)  $\sec^2 x + 2 \sec^2 x \tan x + C$   
(D)  $x \tan x - \ln |\cos x| + C$  (E)  $x \tan x + \ln |\cos x| + C$

30. What is the volume of the solid generated by rotating about the  $x$ -axis the region enclosed by the curve  $y = \sec x$  and the lines  $x = 0$ ,  $y = 0$ , and  $x = \frac{\pi}{3}$ ?

- (A)  $\frac{\pi}{\sqrt{3}}$   
(B)  $\pi$   
(C)  $\pi\sqrt{3}$   
(D)  $\frac{8\pi}{3}$   
(E)  $\pi \ln\left(\frac{1}{2} + \sqrt{3}\right)$

31. If  $s_n = \left( \frac{(5+n)^{100}}{5^{n+1}} \right) \left( \frac{5^n}{(4+n)^{100}} \right)$ , to what number does the sequence  $\{s_n\}$  converge?

- (A)  $\frac{1}{5}$       (B) 1      (C)  $\frac{5}{4}$       (D)  $\left(\frac{5}{4}\right)^{100}$       (E) The sequence does not converge.

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32. If  $\int_a^b f(x)dx = 5$  and  $\int_a^b g(x)dx = -1$ , which of the following must be true?

- I.  $f(x) > g(x)$  for  $a \leq x \leq b$   
II.  $\int_a^b (f(x) + g(x))dx = 4$   
III.  $\int_a^b (f(x)g(x))dx = -5$

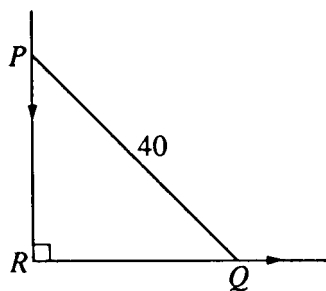
- (A) I only      (B) II only      (C) III only      (D) II and III only      (E) I, II, and III

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33. Which of the following is equal to  $\int_0^\pi \sin x \, dx$ ?

- (A)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$       (B)  $\int_0^\pi \cos x \, dx$       (C)  $\int_{-\pi}^0 \sin x \, dx$   
(D)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx$       (E)  $\int_\pi^{2\pi} \sin x \, dx$





34. In the figure above,  $PQ$  represents a 40-foot ladder with end  $P$  against a vertical wall and end  $Q$  on level ground. If the ladder is slipping down the wall, what is the distance  $RQ$  at the instant when  $Q$  is moving along the ground  $\frac{3}{4}$  as fast as  $P$  is moving down the wall?

(A)  $\frac{6}{5}\sqrt{10}$       (B)  $\frac{8}{5}\sqrt{10}$       (C)  $\frac{80}{\sqrt{7}}$       (D) 24      (E) 32

35. If  $F$  and  $f$  are differentiable functions such that  $F(x) = \int_0^x f(t)dt$ , and if  $F(a) = -2$  and  $F(b) = -2$  where  $a < b$ , which of the following must be true?

(A)  $f(x) = 0$  for some  $x$  such that  $a < x < b$ .  
(B)  $f(x) > 0$  for all  $x$  such that  $a < x < b$ .  
(C)  $f(x) < 0$  for all  $x$  such that  $a < x < b$ .  
(D)  $F(x) \leq 0$  for all  $x$  such that  $a < x < b$ .  
(E)  $F(x) = 0$  for some  $x$  such that  $a < x < b$ .

36. Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume?

(A) 3 cm      (B) 10 cm      (C) 20 cm      (D)  $\frac{30}{\pi^2}$  cm      (E)  $\frac{10}{\pi}$  cm

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37. If  $f(x) = \begin{cases} x & \text{for } x \leq 1 \\ \frac{1}{x} & \text{for } x > 1, \end{cases}$  then  $\int_0^e f(x) dx =$

- (A) 0                      (B)  $\frac{3}{2}$                       (C) 2                      (D)  $e$                       (E)  $e + \frac{1}{2}$

38. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

- (A) 343                      (B) 1,343                      (C) 1,367                      (D) 1,400                      (E) 2,057

39. If  $\frac{dy}{dx} = \frac{1}{x}$ , then the average rate of change of  $y$  with respect to  $x$  on the closed interval  $[1, 4]$  is

- (A)  $-\frac{1}{4}$                       (B)  $\frac{1}{2} \ln 2$                       (C)  $\frac{2}{3} \ln 2$                       (D)  $\frac{2}{5}$                       (E) 2

40. Let  $R$  be the region in the first quadrant enclosed by the  $x$ -axis and the graph of  $y = \ln(1 + 2x - x^2)$ . If Simpson's Rule with 2 subintervals is used to approximate the area of  $R$ , the approximation is

- (A) 0.462                      (B) 0.693                      (C) 0.924                      (D) 0.986                      (E) 1.850

41. Let  $f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$ . At what value of  $x$  is  $f(x)$  a minimum?

- (A) For no value of  $x$                       (B)  $\frac{1}{2}$                       (C)  $\frac{3}{2}$                       (D) 2                      (E) 3

42.  $\lim_{x \rightarrow 0} (1 + 2x)^{\csc x} =$

- (A) 0                      (B) 1                      (C) 2                      (D)  $e$                       (E)  $e^2$

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43. The coefficient of  $x^6$  in the Taylor series expansion about  $x = 0$  for  $f(x) = \sin(x^2)$  is

- (A)  $-\frac{1}{6}$       (B) 0      (C)  $\frac{1}{120}$       (D)  $\frac{1}{6}$       (E) 1
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44. If  $f$  is continuous on the interval  $[a, b]$ , then there exists  $c$  such that  $a < c < b$  and  $\int_a^b f(x)dx =$

- (A)  $\frac{f(c)}{b-a}$     (B)  $\frac{f(b)-f(a)}{b-a}$     (C)  $f(b)-f(a)$     (D)  $f'(c)(b-a)$     (E)  $f(c)(b-a)$
- 

45. If  $f(x) = \sum_{k=1}^{\infty} (\sin^2 x)^k$ , then  $f(1)$  is

- (A) 0.369      (B) 0.585      (C) 2.400      (D) 2.426      (E) 3.426